

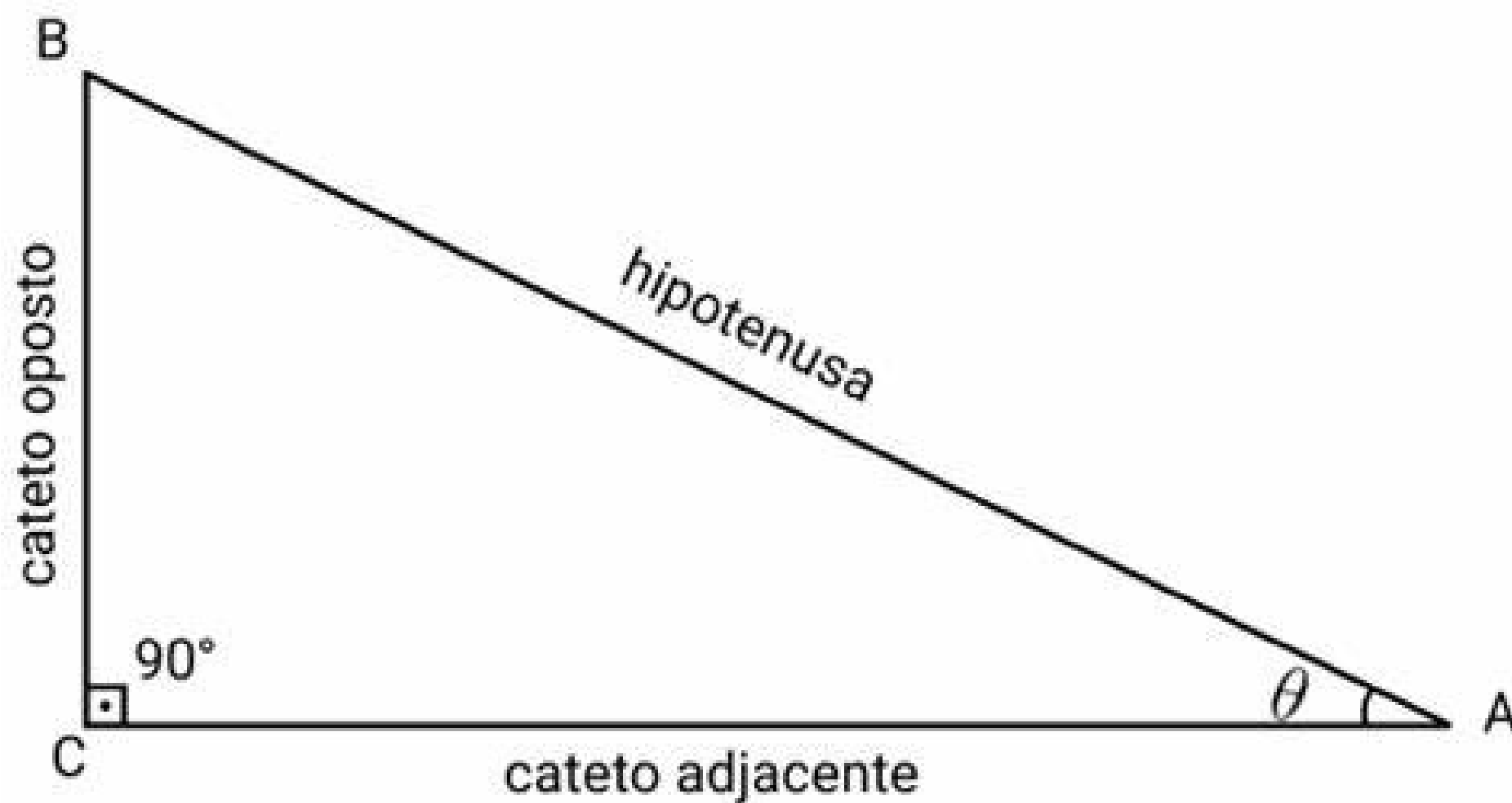
TRIGONOMETRIA

# AULAS 08 e 09



Diana Caires, Fernanda Tenorio e Mateus Marques

# Trigonometria no triângulo retângulo

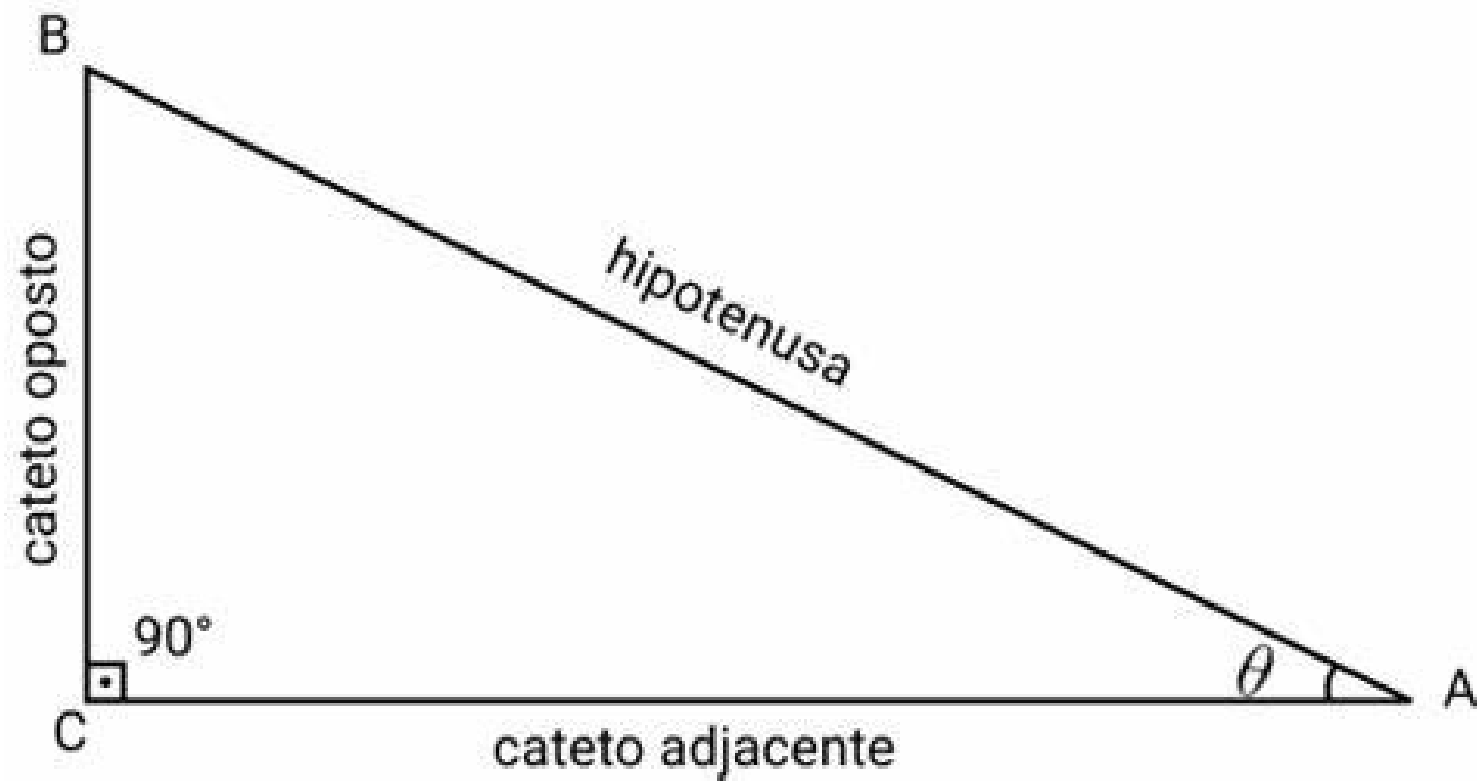


$$\text{sen } \theta = \frac{CO}{h}$$

$$\text{tg } \theta = \frac{CO}{CA}$$

$$\text{cos } \theta = \frac{CA}{h}$$

# Cotangente, Secante e Cossecante



$$\cotg \theta = \frac{CA}{CO} = \frac{h^* \cos \theta}{h^* \sin \theta} = \frac{\cos \theta}{\sin \theta}$$

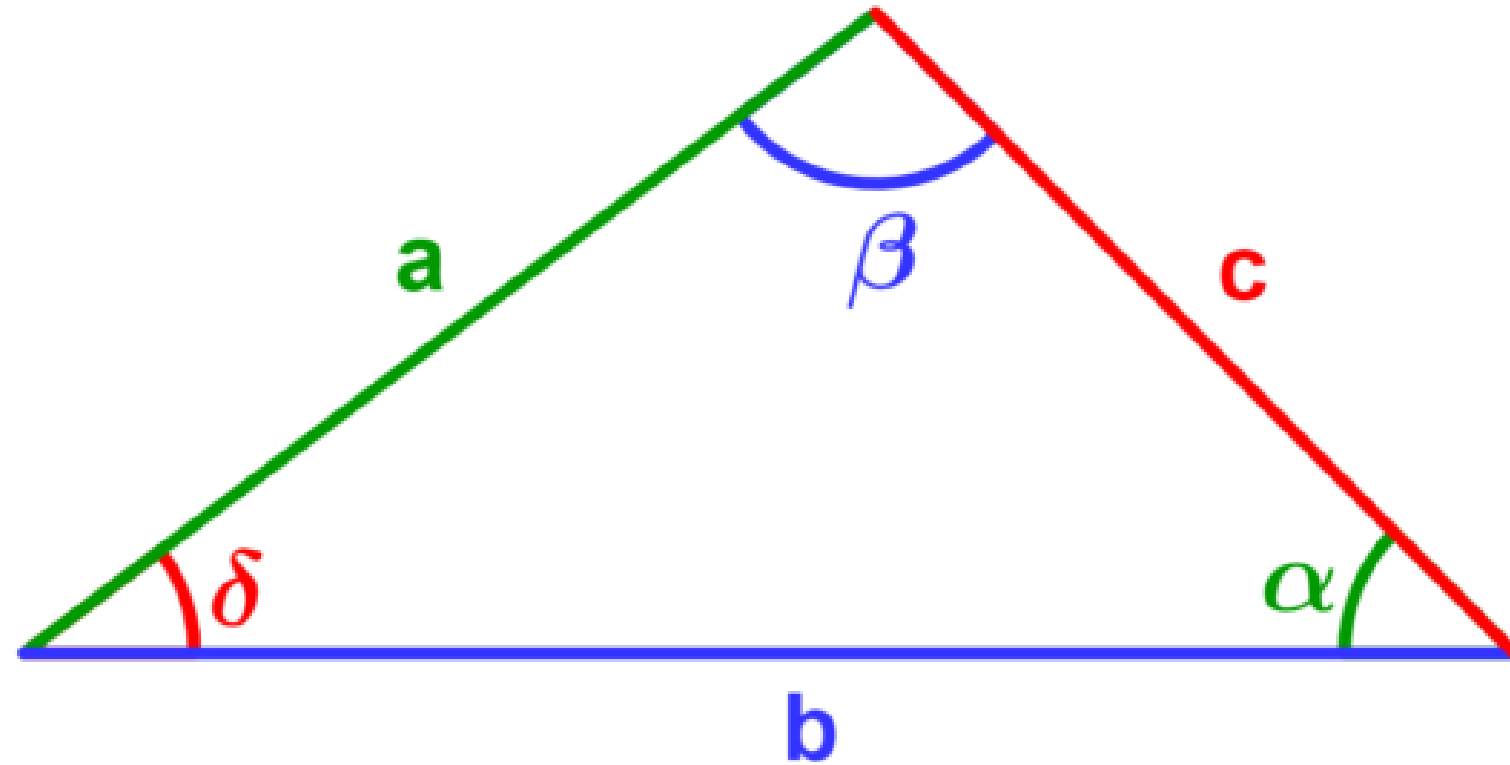
$$\sec \theta = \frac{h}{CA} = \frac{h}{h^* \cos \theta} = \frac{1}{\cos \theta}$$

$$\operatorname{cosec} \theta = \frac{h}{CO} = \frac{h}{h^* \sin \theta} = \frac{1}{\sin \theta}$$

$$\sin \theta = \frac{CO}{h} \longrightarrow CO = h^* \sin \theta$$

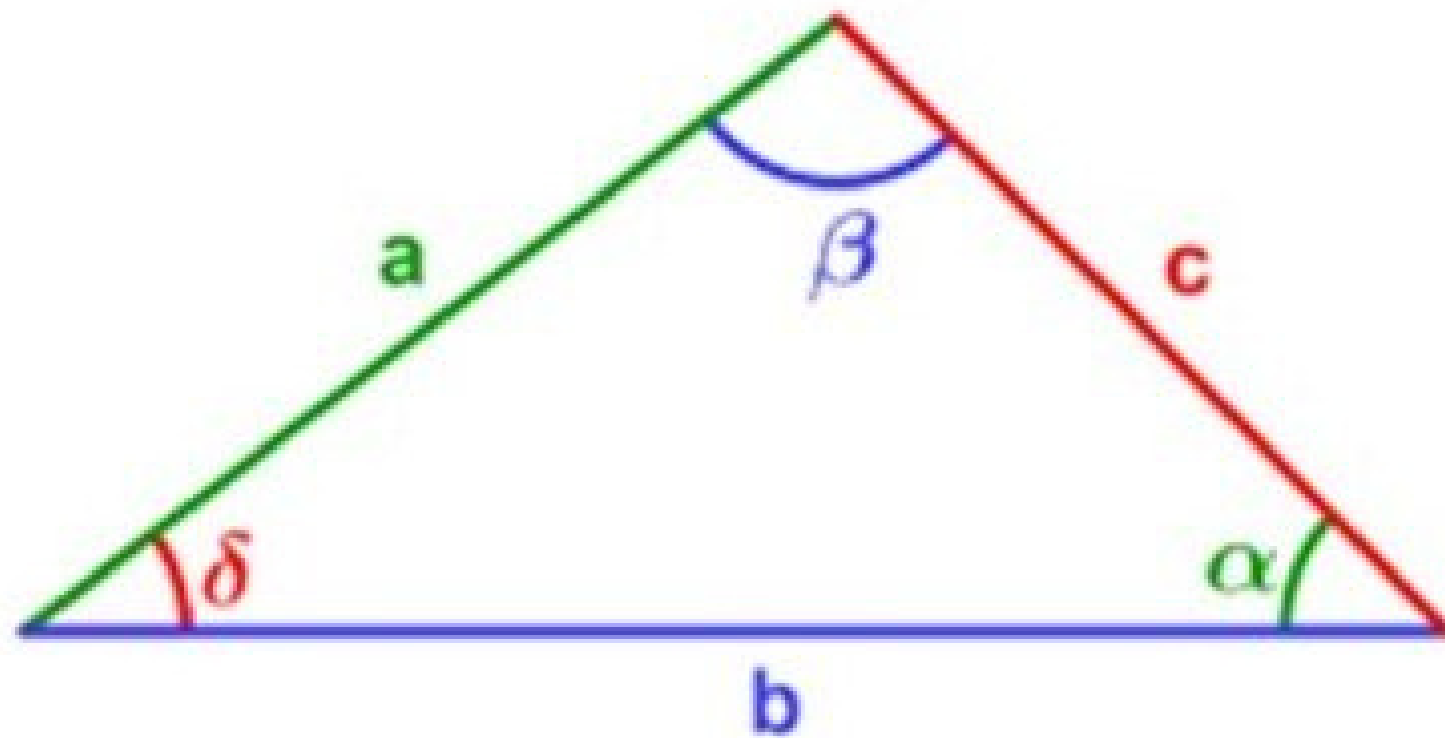
$$\cos \theta = \frac{CA}{h} \longrightarrow CA = h^* \cos \theta$$

# Lei dos Senos



$$\frac{a}{\text{sen } \alpha} = \frac{b}{\text{sen } \beta} = \frac{c}{\text{sen } \delta}$$

# Lei dos Cossenos



$$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos \alpha$$

$$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos \beta$$

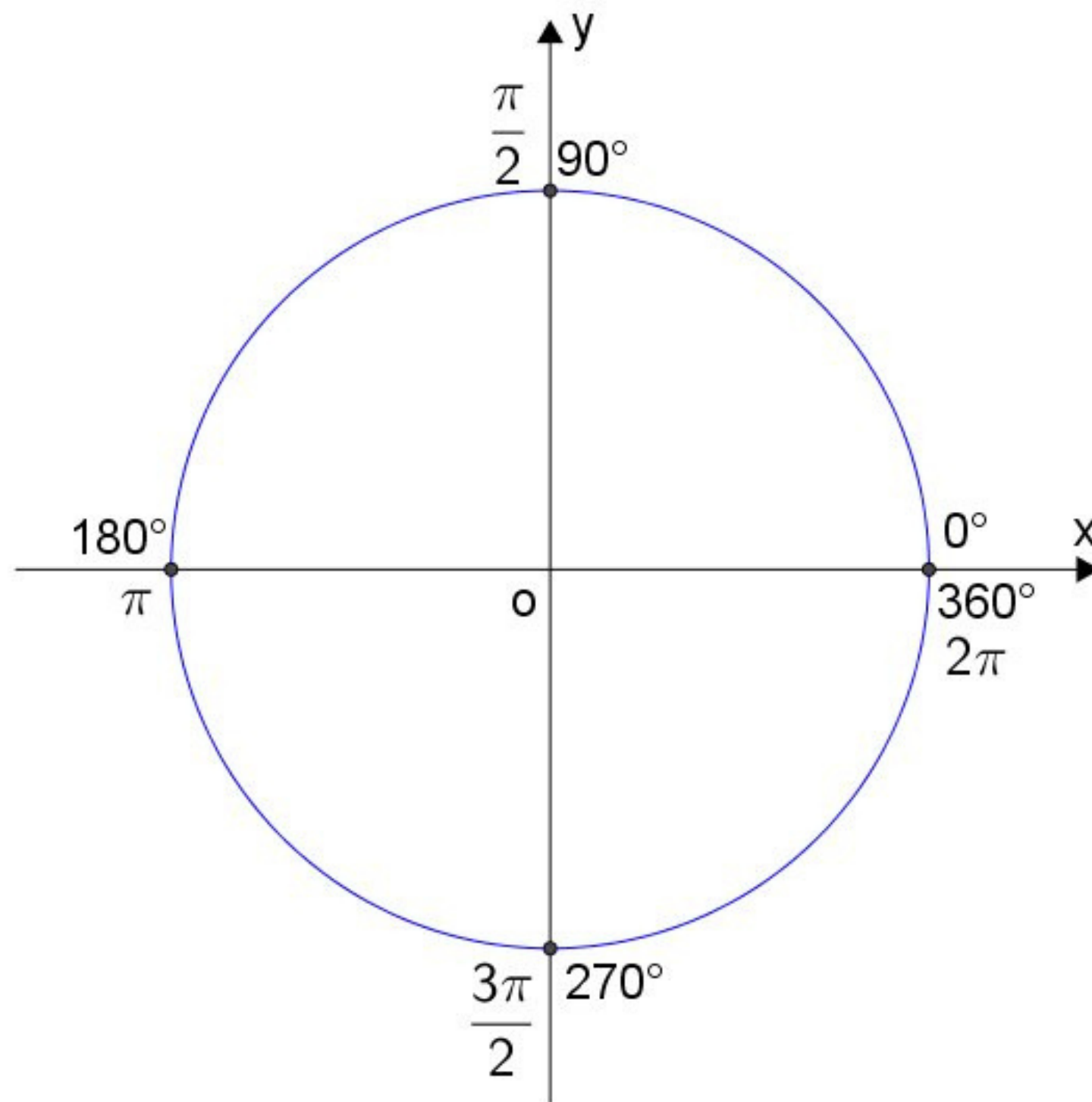
$$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos \gamma$$

# Ângulos notáveis

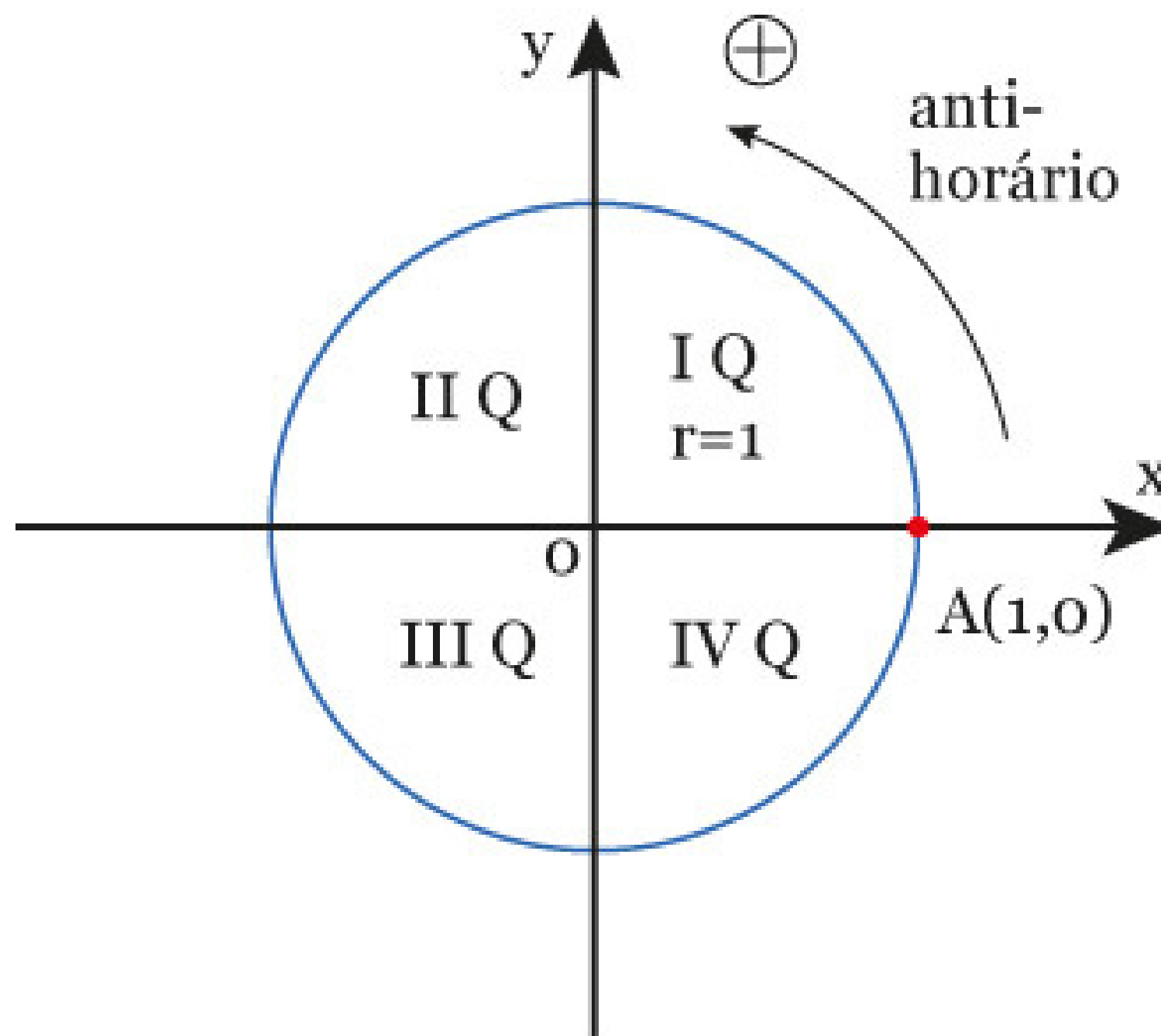


	<b>30<sup>o</sup></b>	<b>45<sup>o</sup></b>	<b>60<sup>o</sup></b>
<b>SENO</b>	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
<b>COSSENO</b>	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
<b>TANGENTE</b>	$\frac{\sqrt{3}}{3}$	<b>1</b>	$\sqrt{3}$

# Círculo trigonométrico



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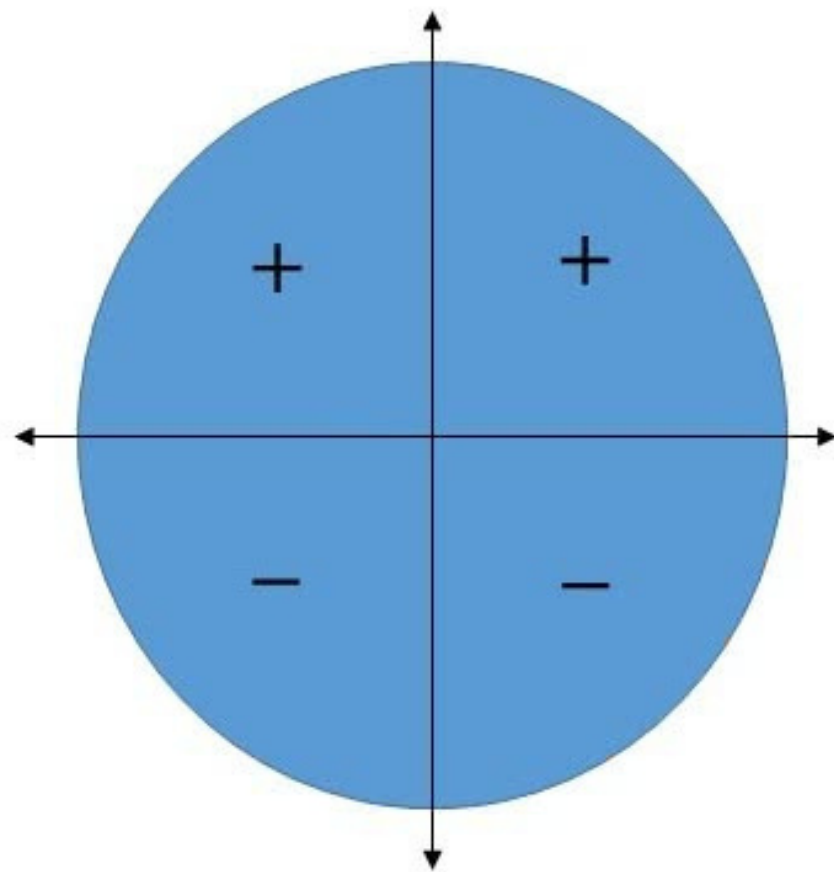




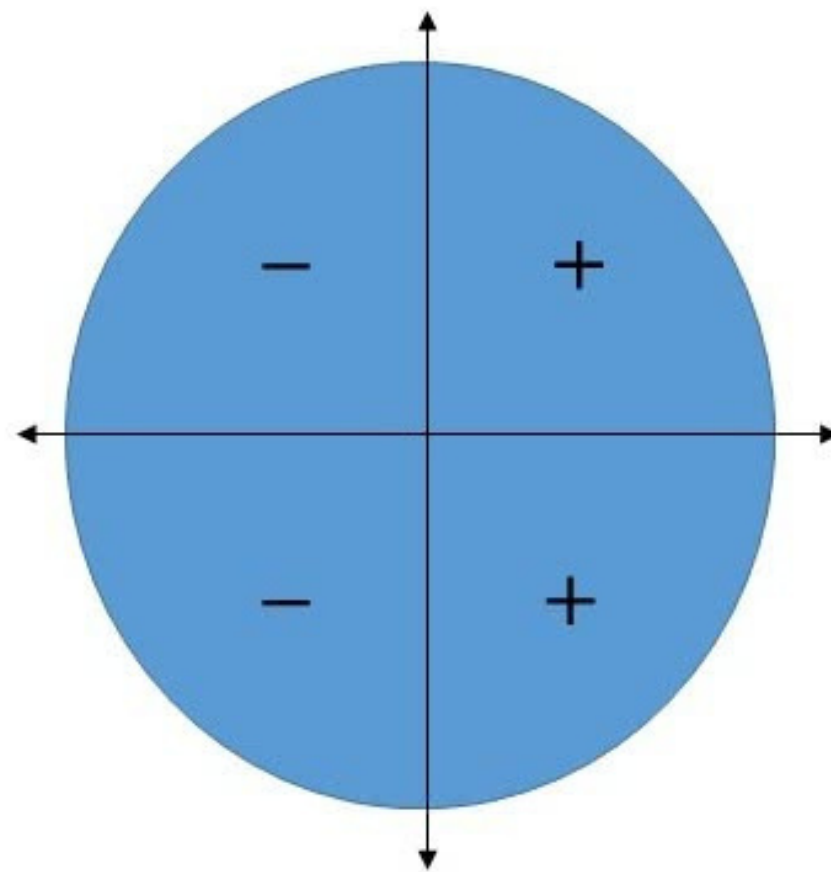
# Círculo trigonométrico



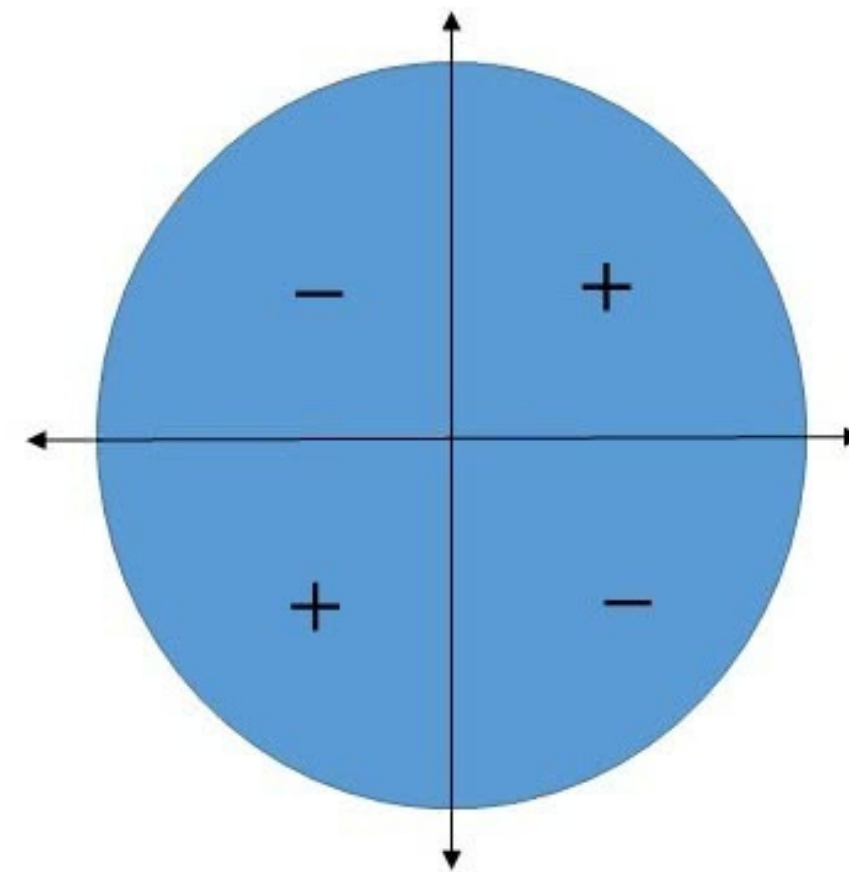
**Seno**



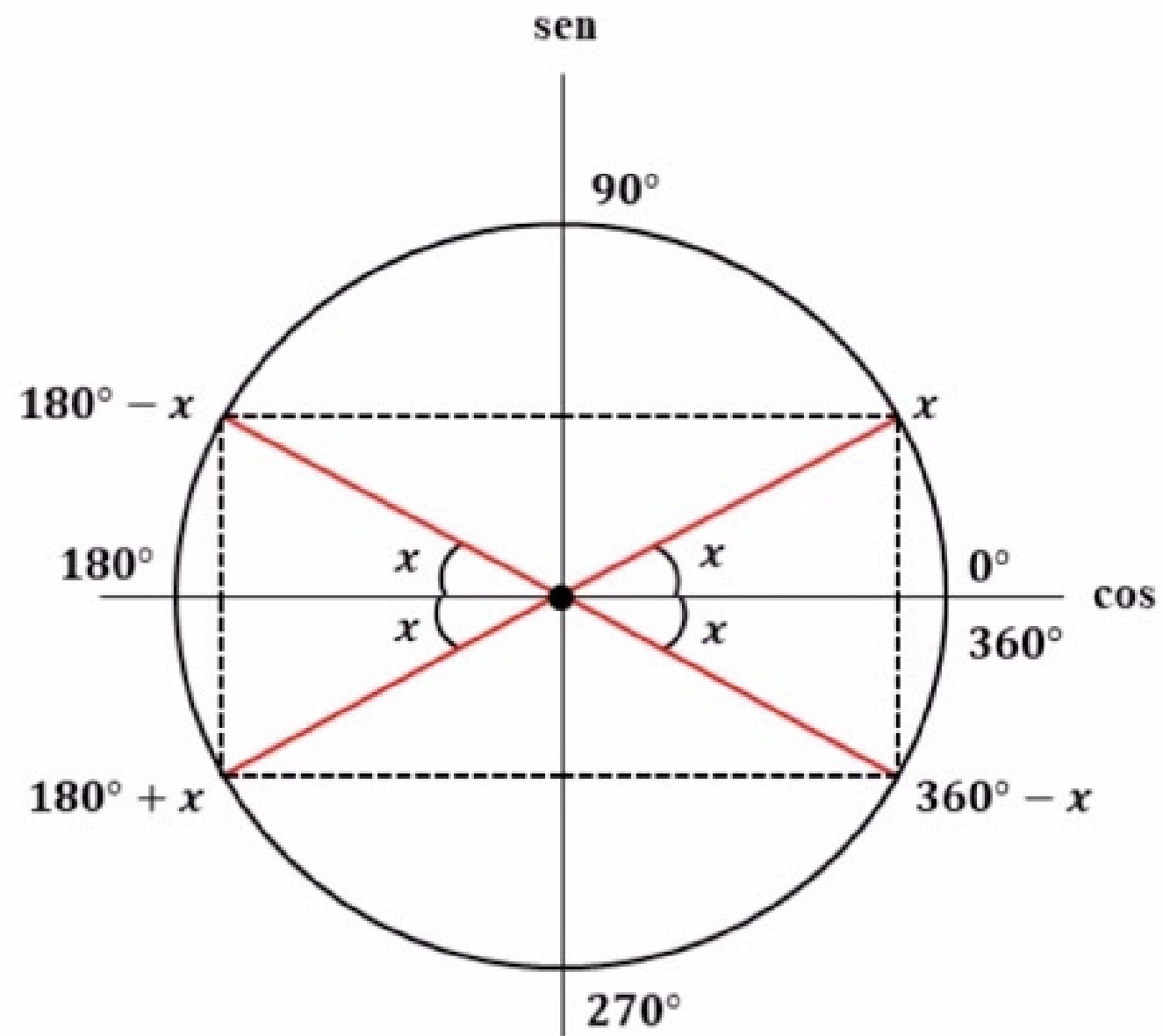
**Cosseno**



**Tangente**



# Redução ao primeiro quadrante



## DO 2° AO 1° QUADRANTE

$180^\circ - x$  e  $x \rightarrow$  suplementares

$$\text{sen}(180^\circ - x) = \text{sen } x$$

$$\text{cos}(180^\circ - x) = -\text{cos } x$$

## DO 3° AO 1° QUADRANTE

$180^\circ + x$  e  $x \rightarrow$  explementares

$$\text{sen}(180^\circ + x) = -\text{sen } x$$

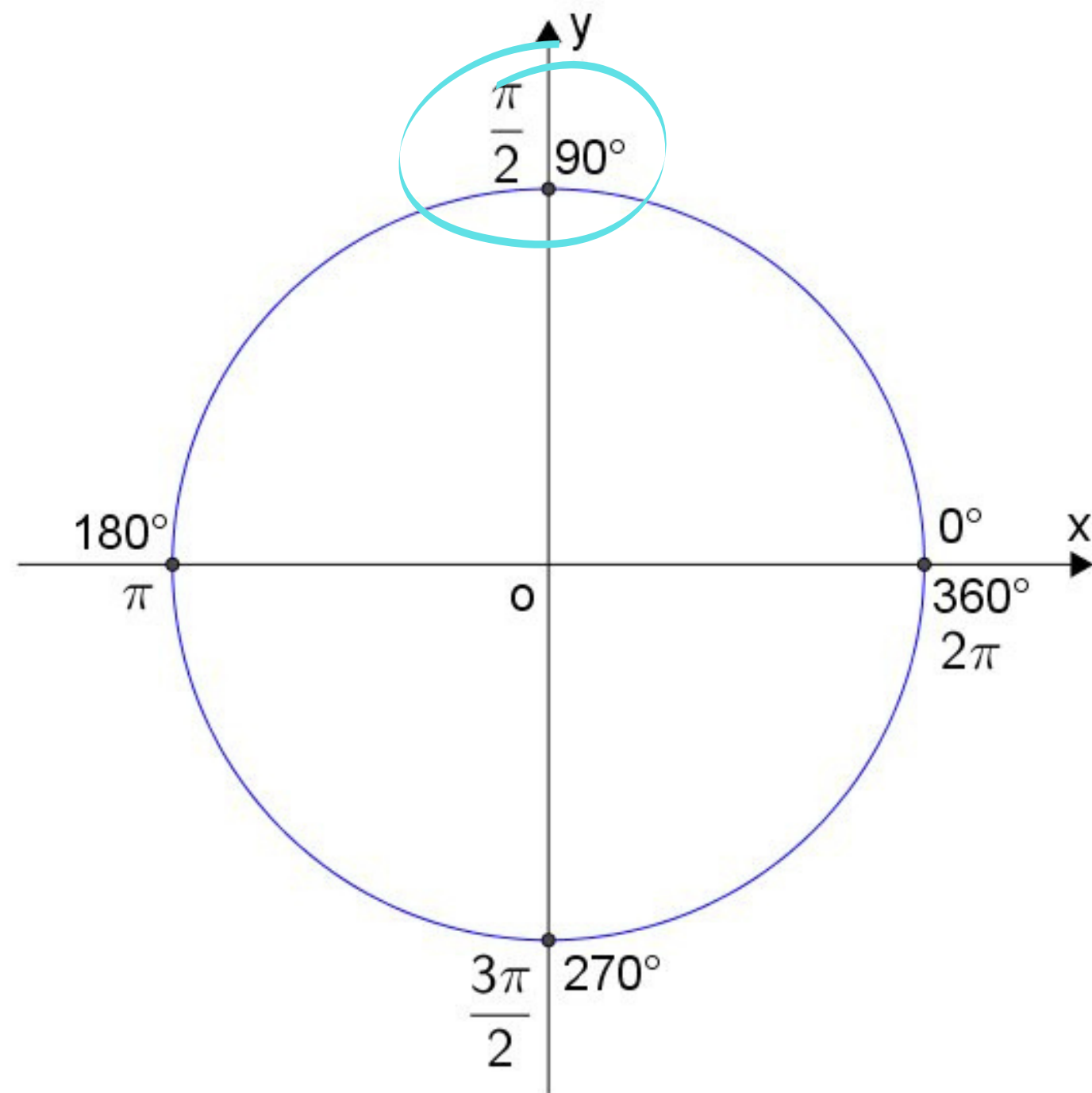
$$\text{cos}(180^\circ + x) = -\text{cos } x$$

## DO 4° AO 1° QUADRANTE

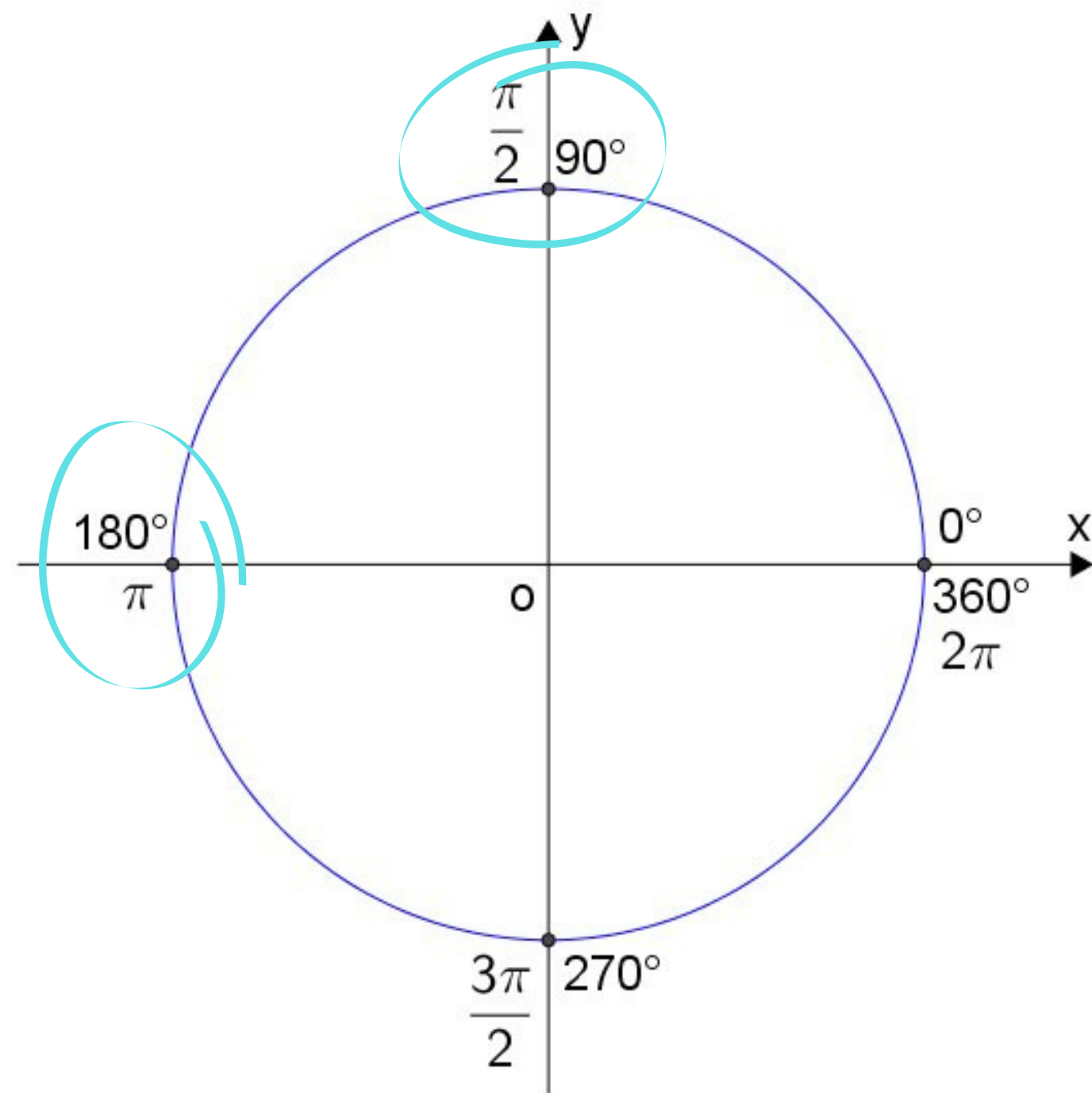
$360^\circ - x$  e  $x \rightarrow$  replementares

$$\text{sen}(360^\circ - x) = -\text{sen } x$$

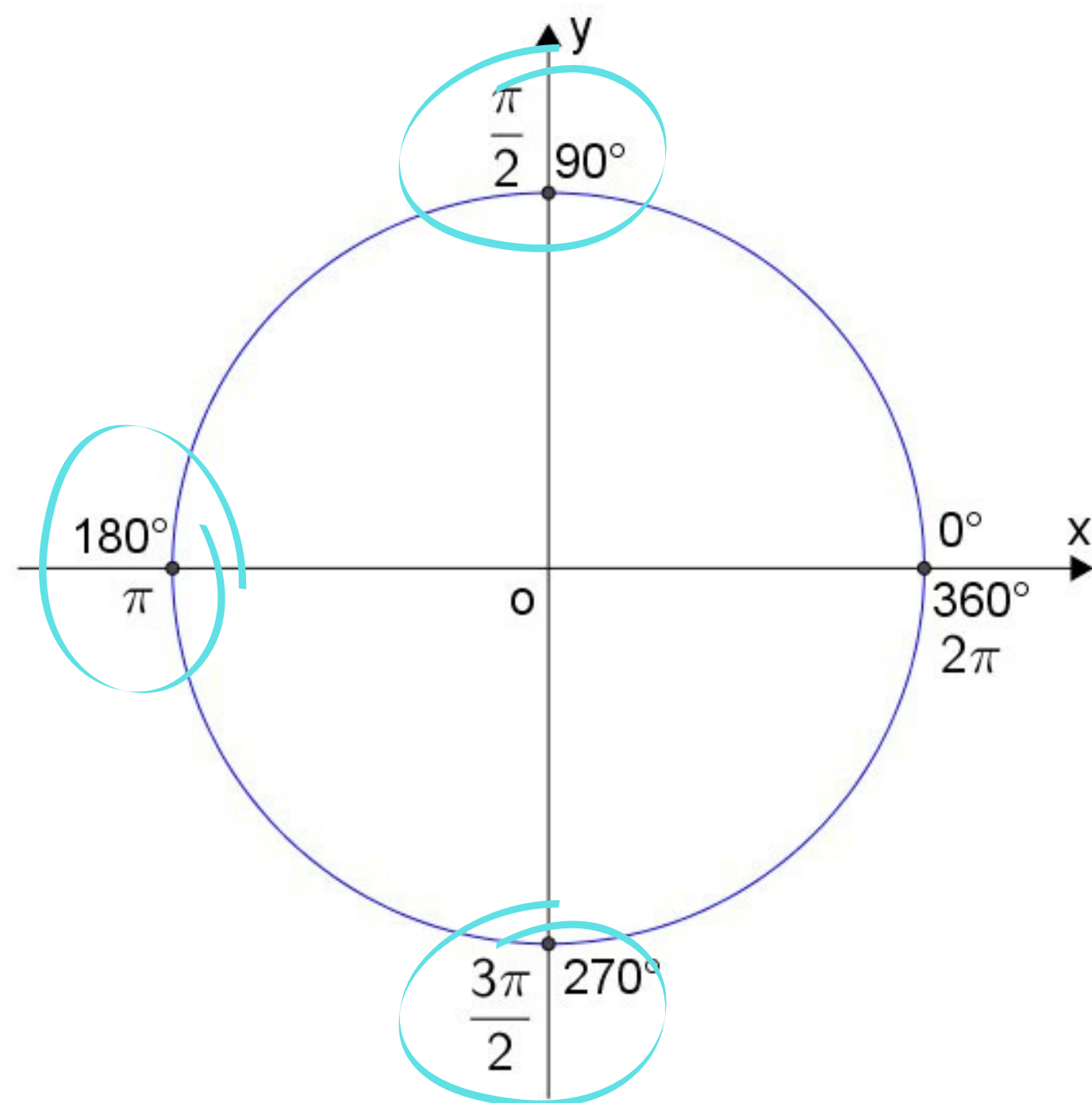
# Transformação de graus para radianos



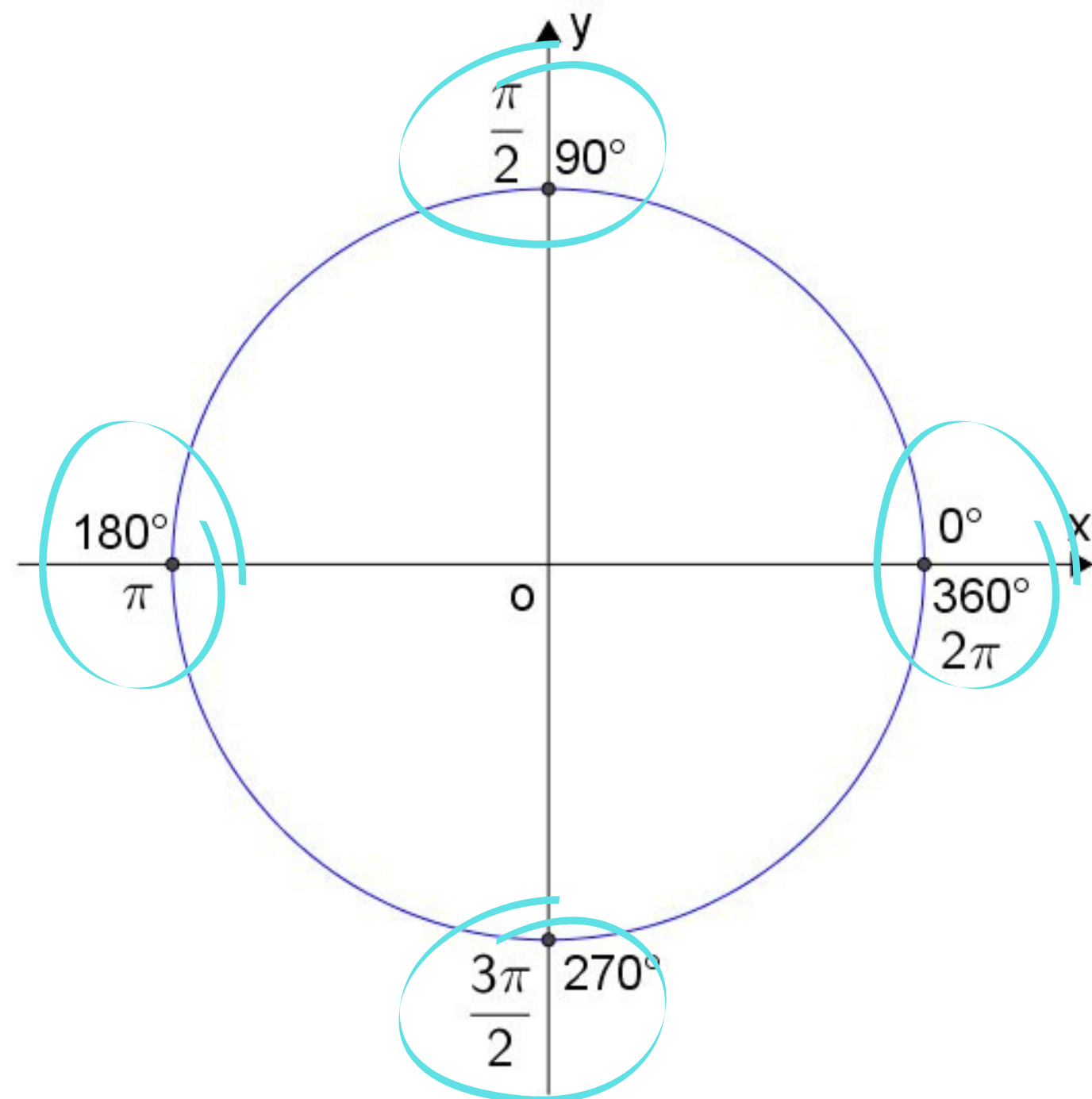
# Transformação de graus para radianos



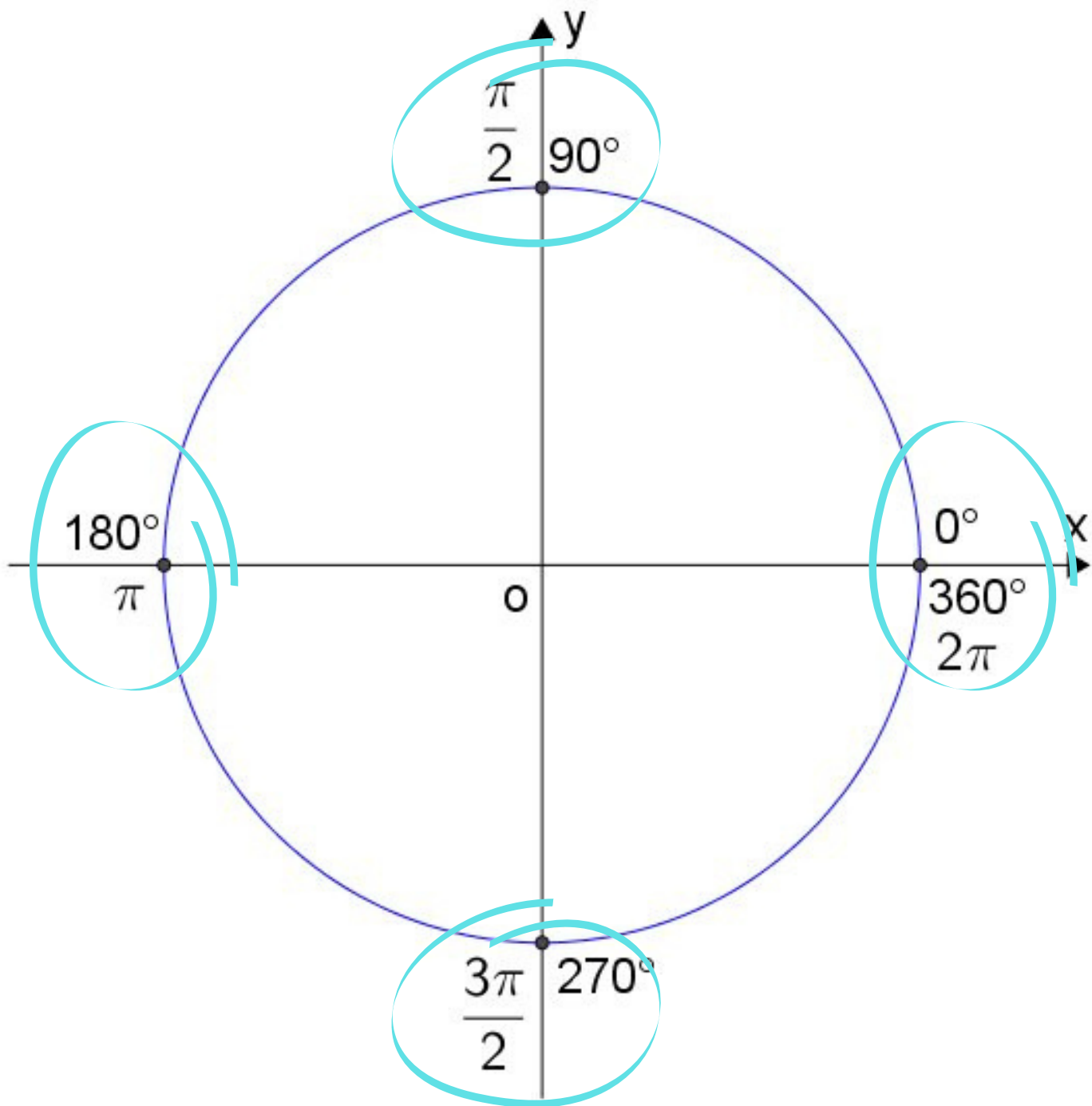
# Transformação de graus para radianos



# Transformação de graus para radianos

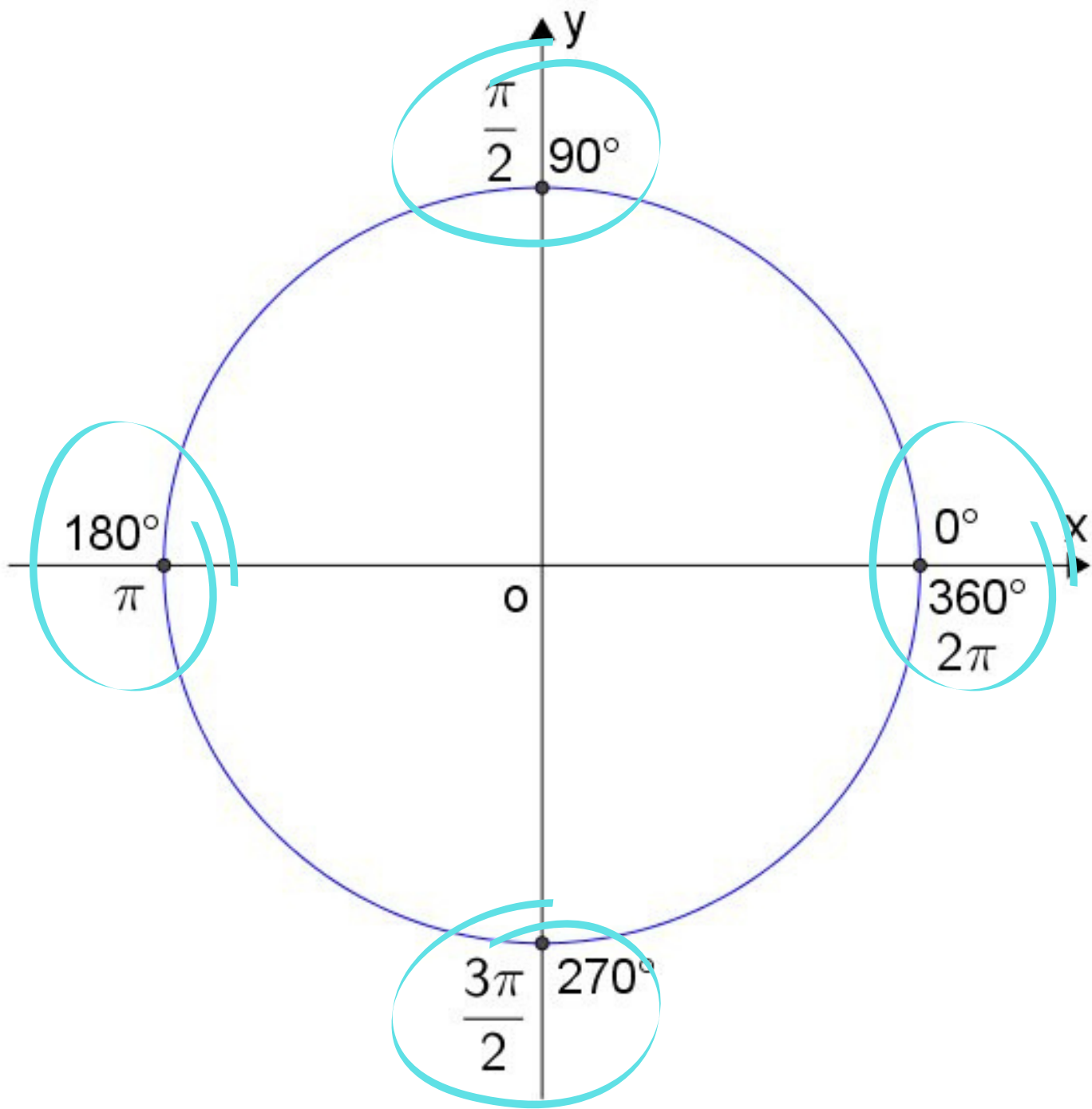


# Transformação de graus para radianos



$$180^\circ \text{ ——— } \pi$$
$$\text{ângulo} \text{ ——— } x$$
$$\text{qualquer} \text{ ——— } x$$

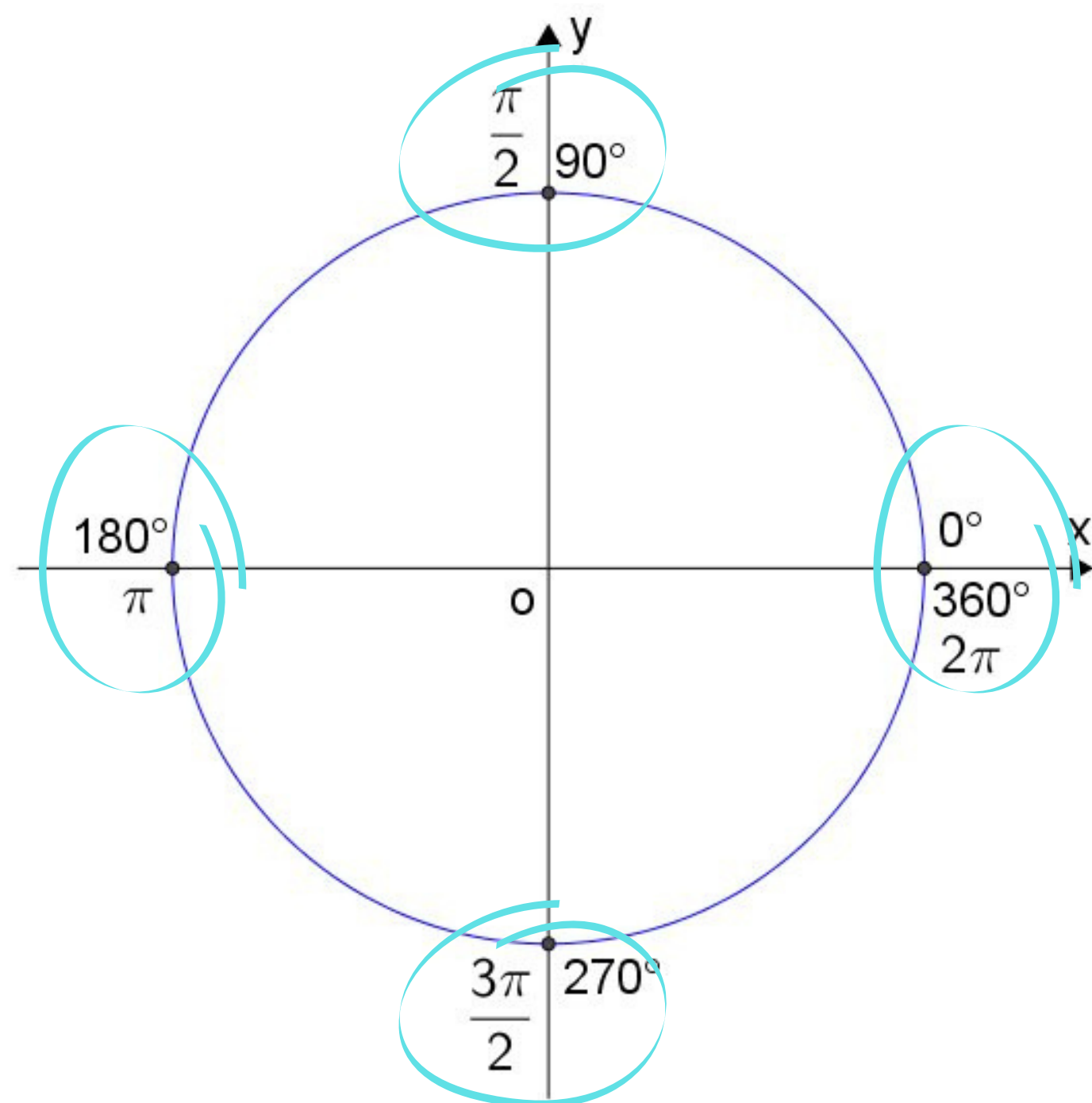
# Transformação de graus para radianos



~~$180^\circ \quad \text{---} \quad \pi$~~   
 ~~$\text{ângulo qualquer} \quad \text{---} \quad x$~~



# Transformação de graus para radianos



~~$180^\circ \text{ --- } \pi$~~   
 ~~$\text{ângulo qualquer} \text{ --- } x$~~

$$x = \frac{\text{ângulo qualquer} \cdot \pi}{180^\circ}$$

# Identidade trigonométrica fundamental



$$\text{sen}^2 \alpha + \text{cos}^2 \alpha = 1$$

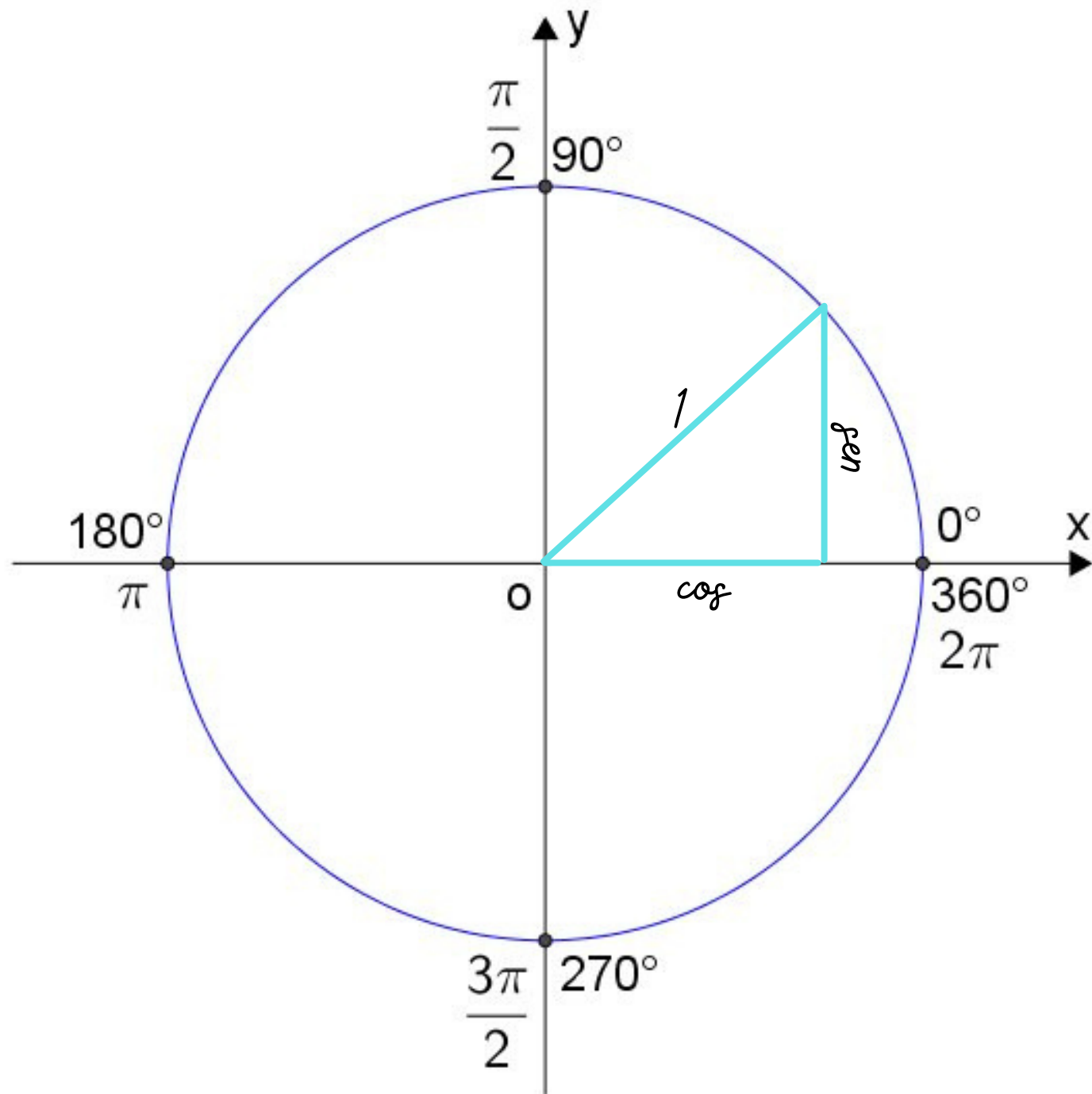
# Identidade trigonométrica fundamental



$$\text{sen}^2 \alpha + \text{cos}^2 \alpha = 1$$

Obs.: surge a partir do círculo trigonométrico!

# Identidade trigonométrica fundamental



$$\text{sen}^2 \alpha + \text{cos}^2 \alpha = 1$$

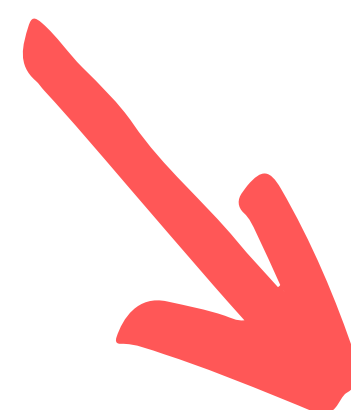
# Identidades trigonométricas complementares



$$\text{sen}^2 \alpha + \text{cos}^2 \alpha = 1$$



$$\text{tg}^2 \alpha + 1 = \text{sec}^2 \alpha$$



$$\text{ctg}^2 \alpha + 1 = \text{csec}^2 \alpha$$

# Soma e Subtração de Arcos



$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \sin \beta \cos \alpha$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

# Arco Duplo



$$\sin(2\alpha) = 2 \sin \alpha \cos \alpha$$

$$\cos(2\alpha) = \cos^2 \alpha - \sin^2 \alpha$$

$$\tan(2\alpha) = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

# Arco Metade



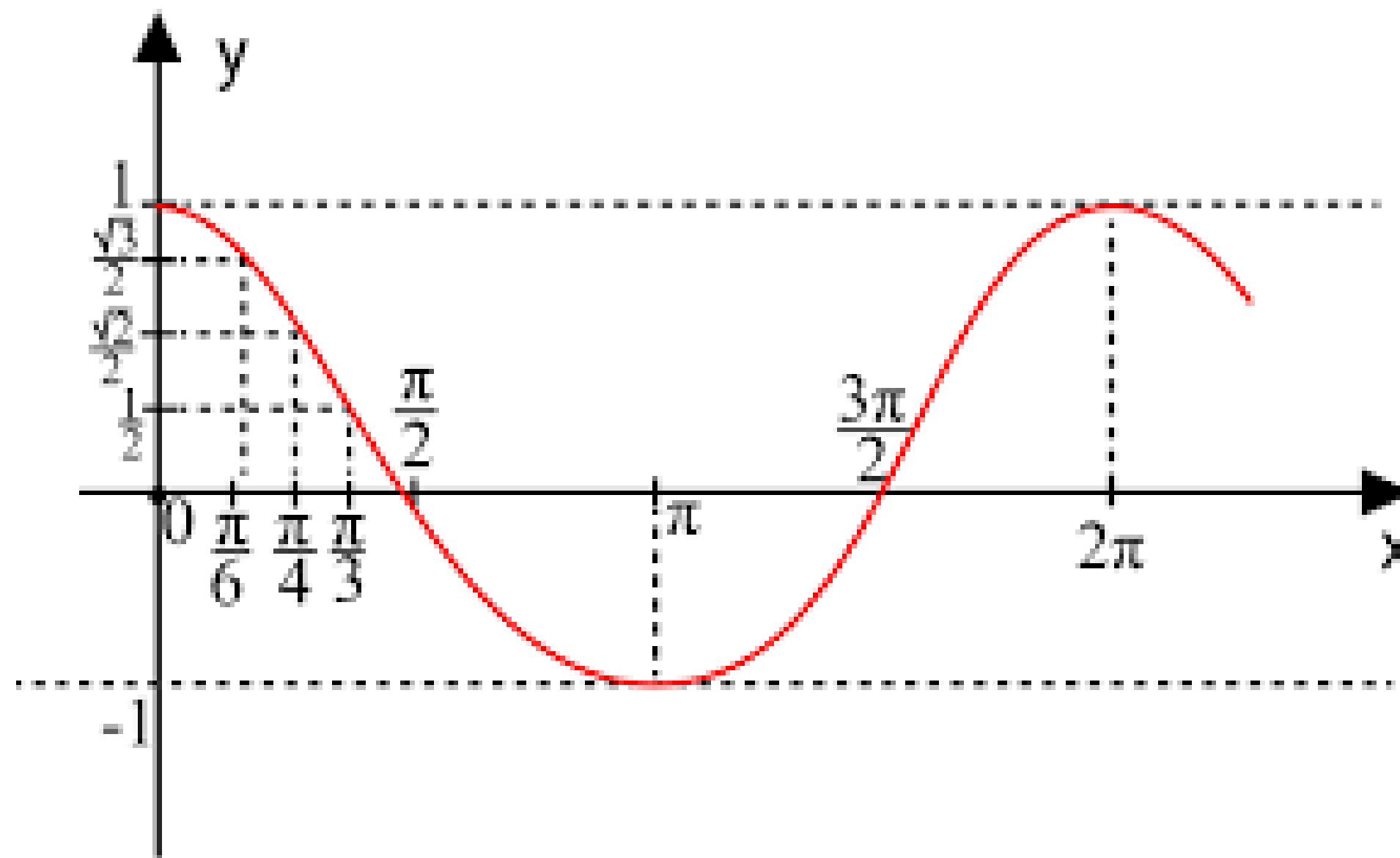
$$\cos x = 2 \cos^2 \frac{x}{2} - 1 \Rightarrow \boxed{\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}}$$

$$\cos x = 1 - 2 \sin^2 \frac{x}{2} \Rightarrow \boxed{\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}}$$

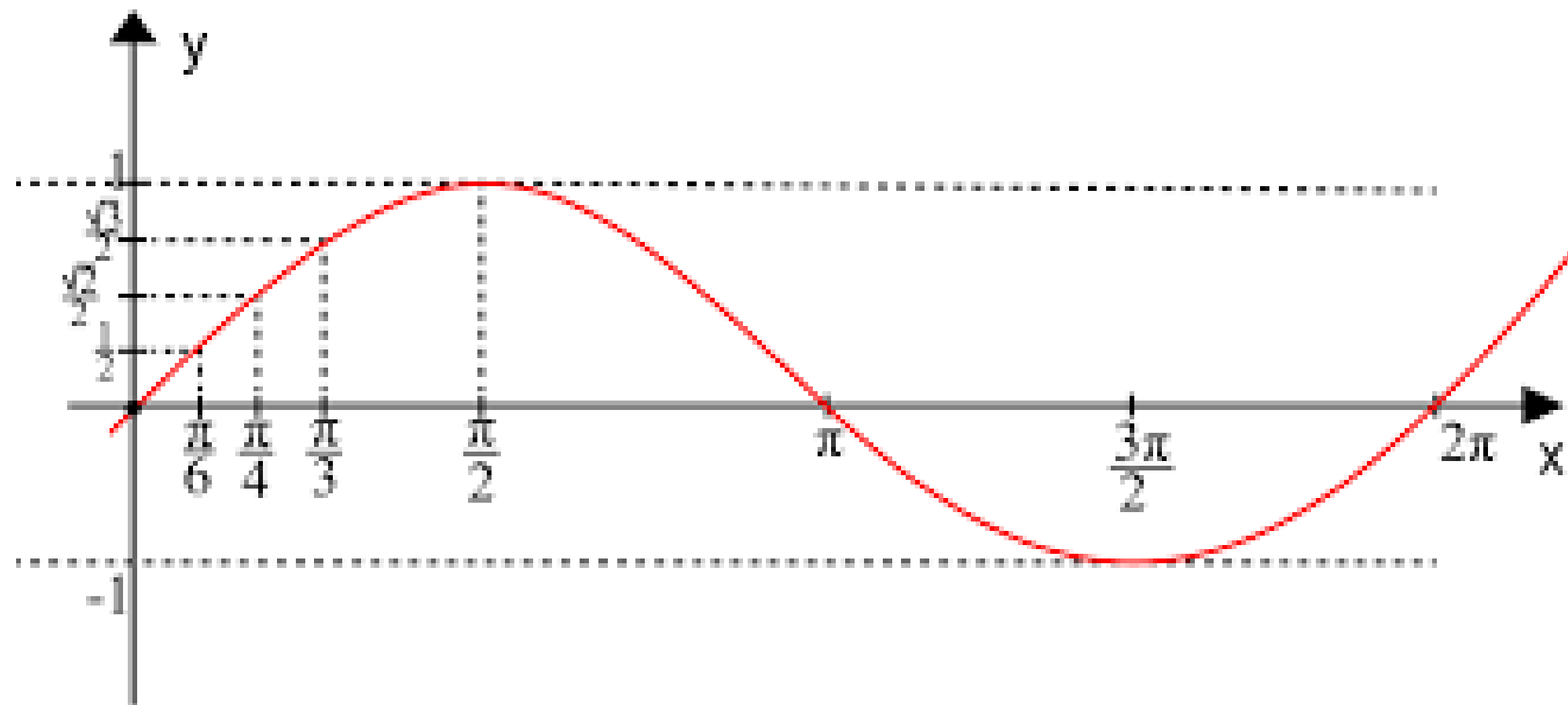
$$\operatorname{tg} \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} \Rightarrow \boxed{\operatorname{tg} \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}}}$$



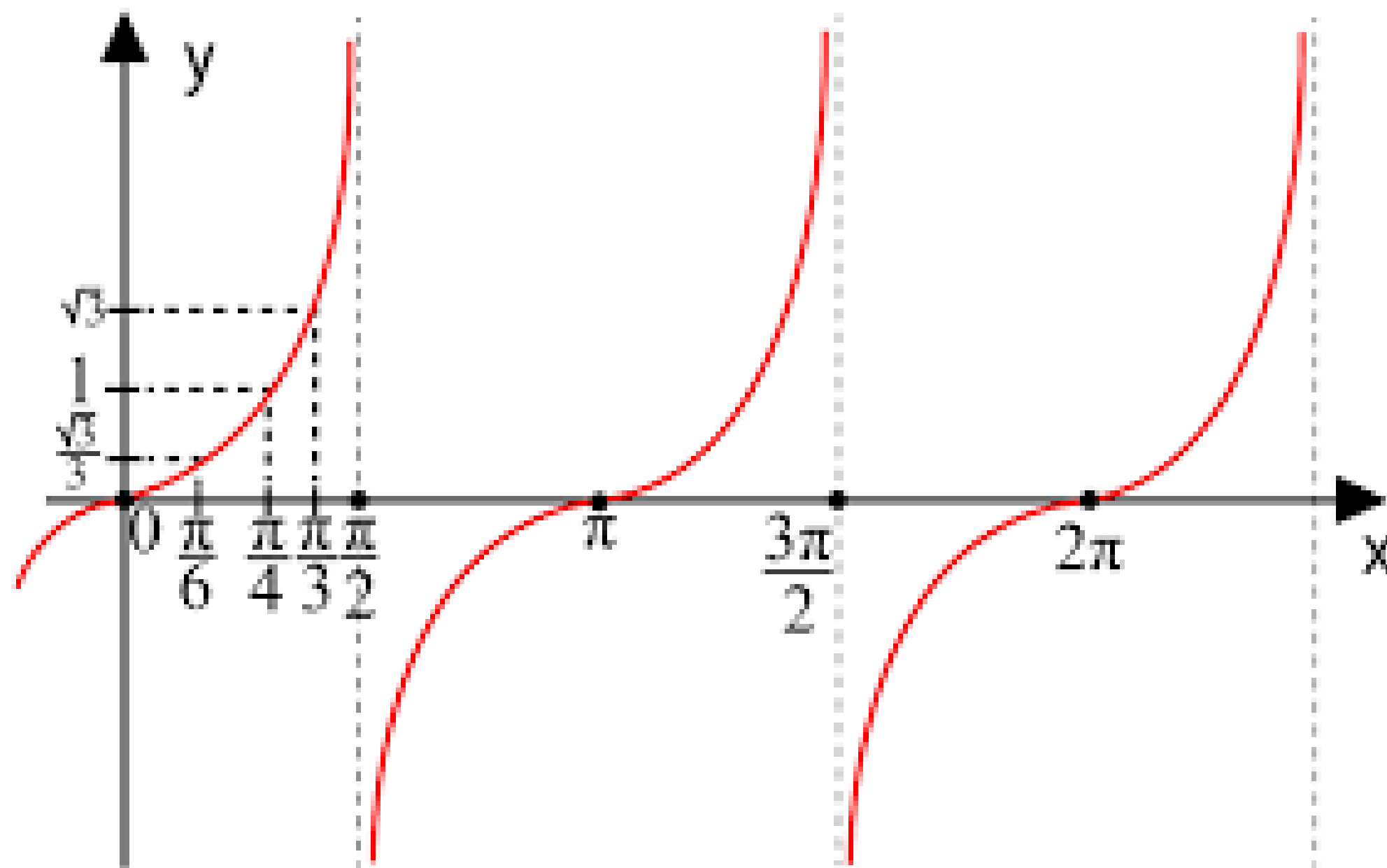
# Funções Trigonométricas: Qual a função?



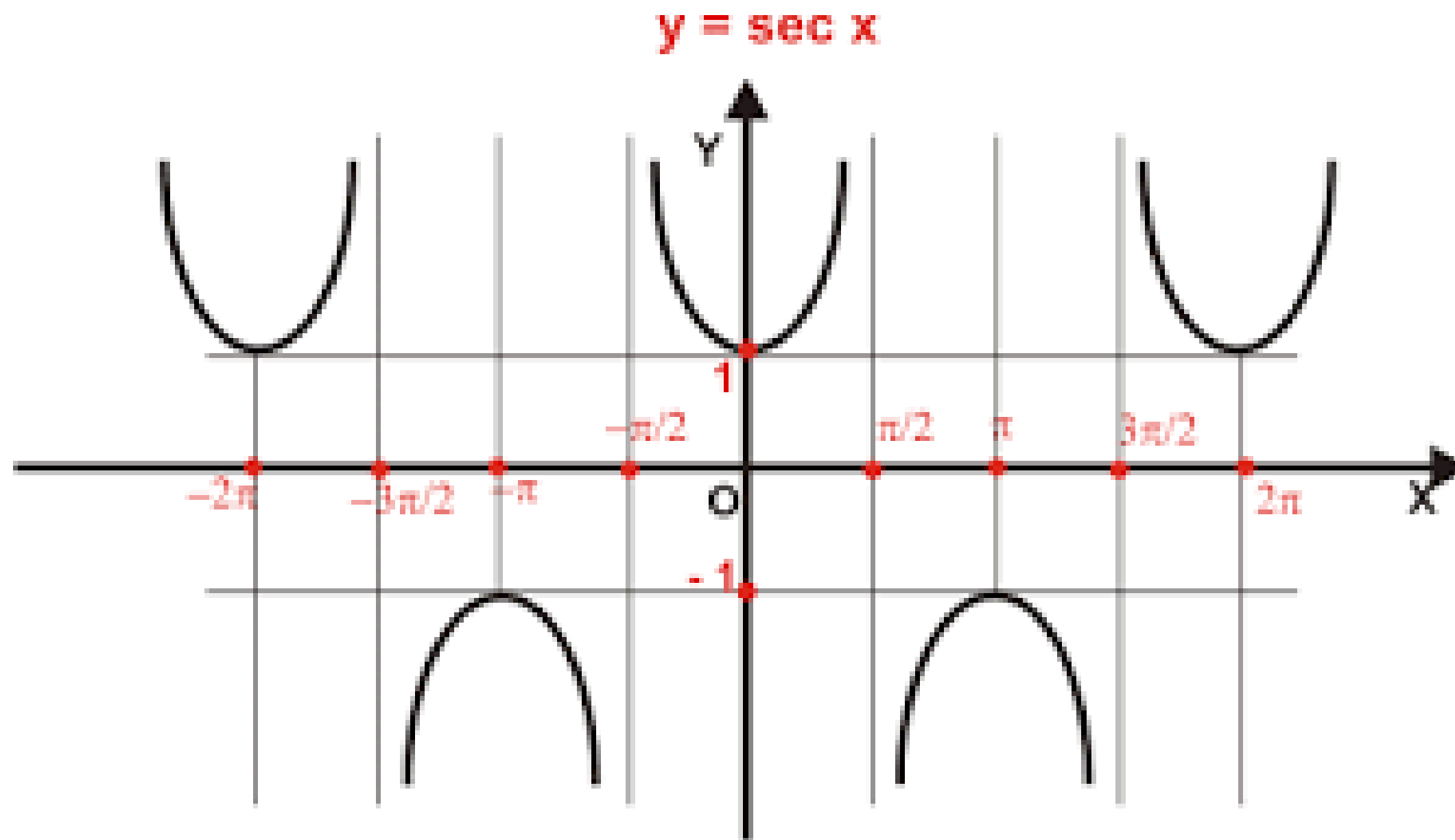
# Funções Trigonométricas: Qual a função?



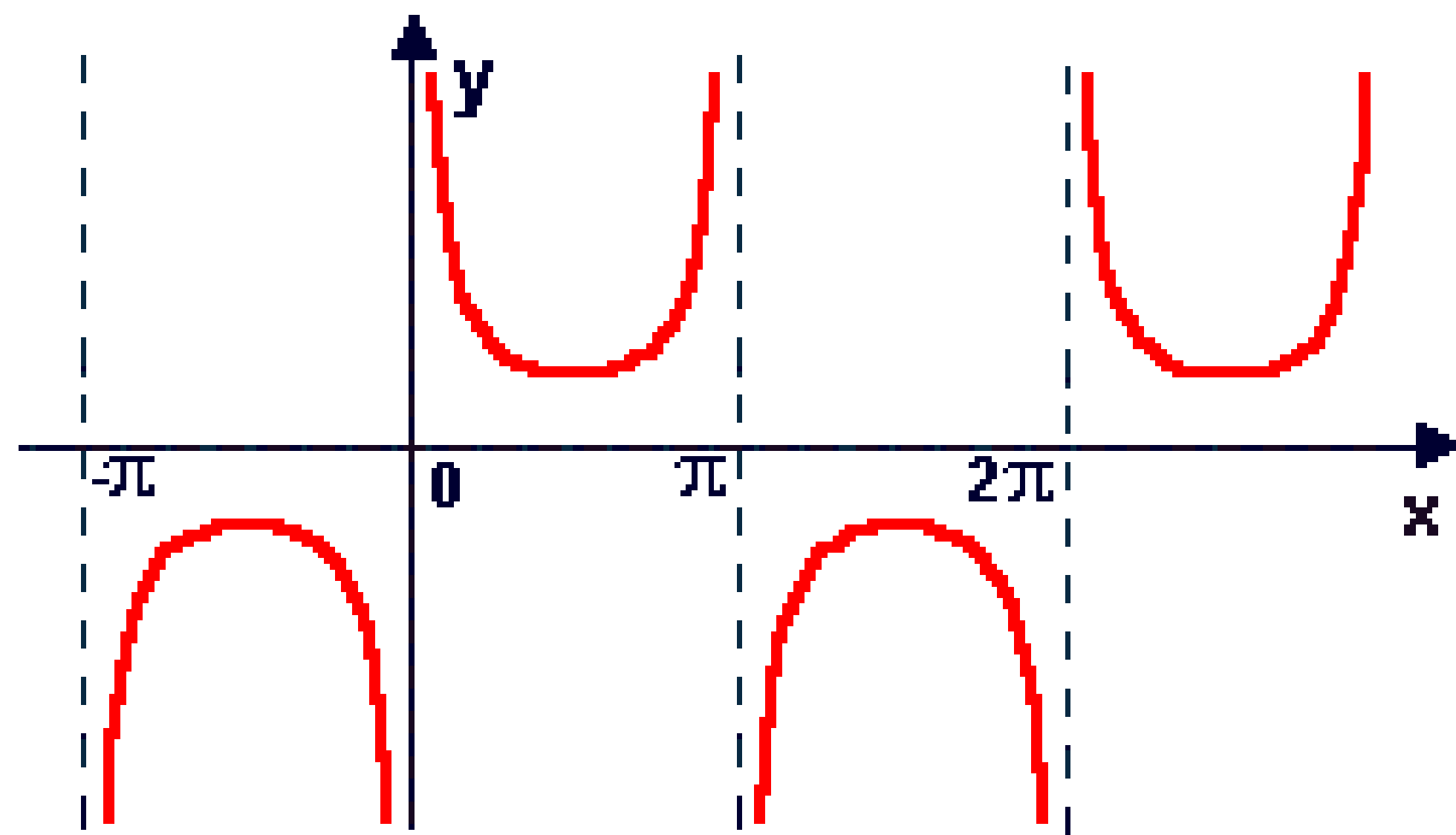
# Funções Trigonométricas: Qual a função?



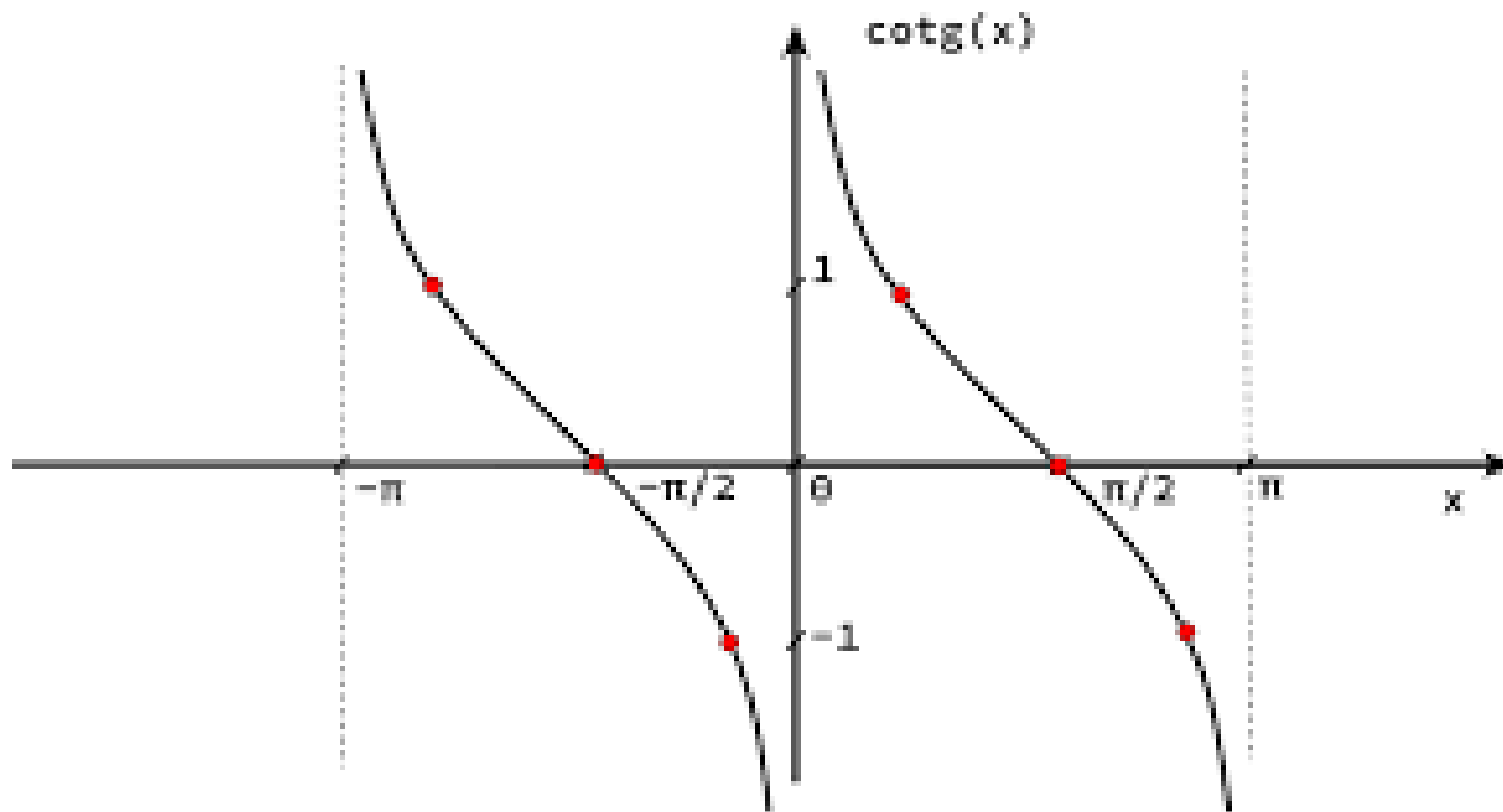
# Função Trigonométrica: Secante



# Função Trigonométrica: Cosecante



# Função Trigonométrica: Cotangente



# Funções trigonométricas inversas

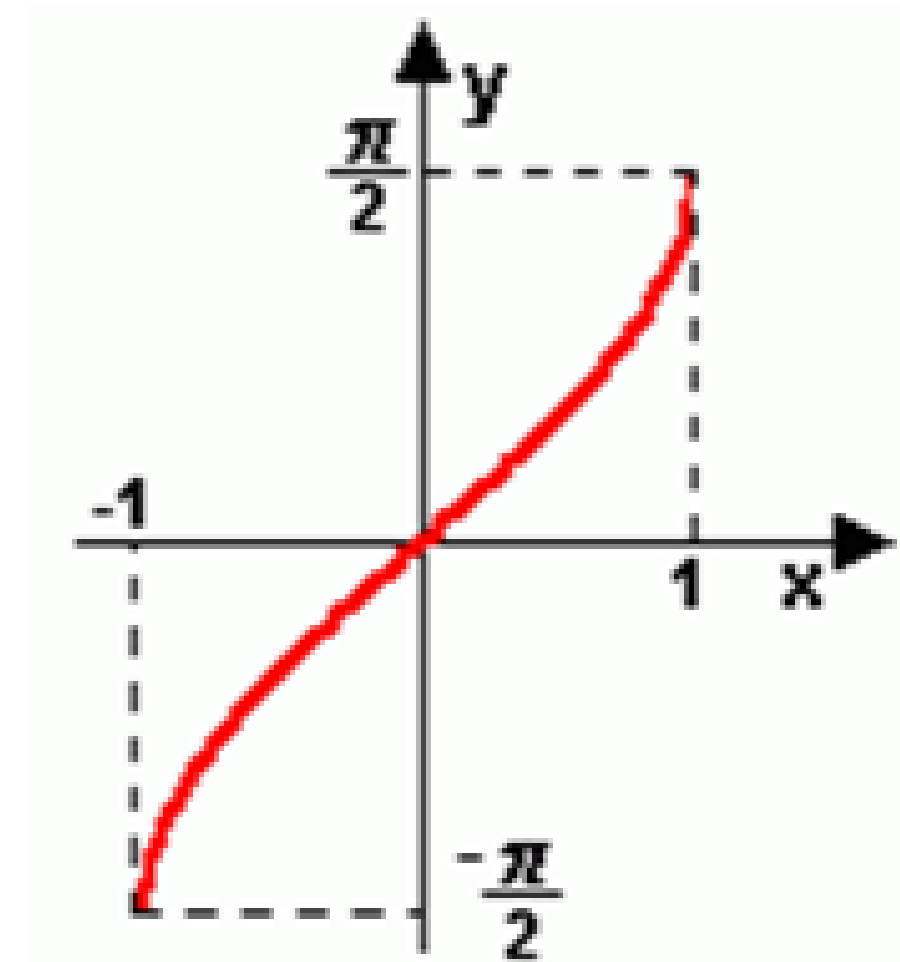
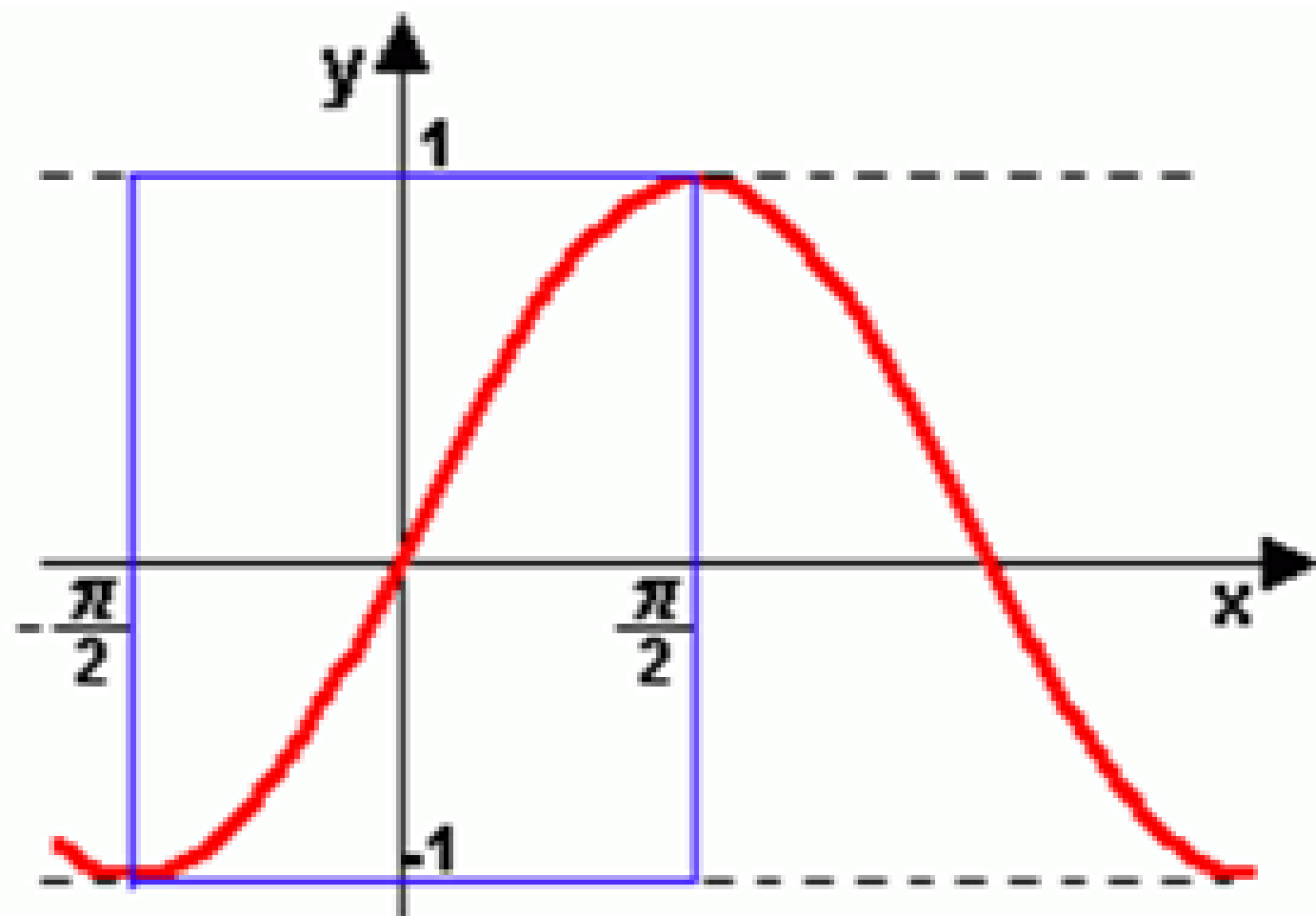


Mas o que é uma função inversa?

A função inversa faz exatamente o inverso da função  $f(x)$ .

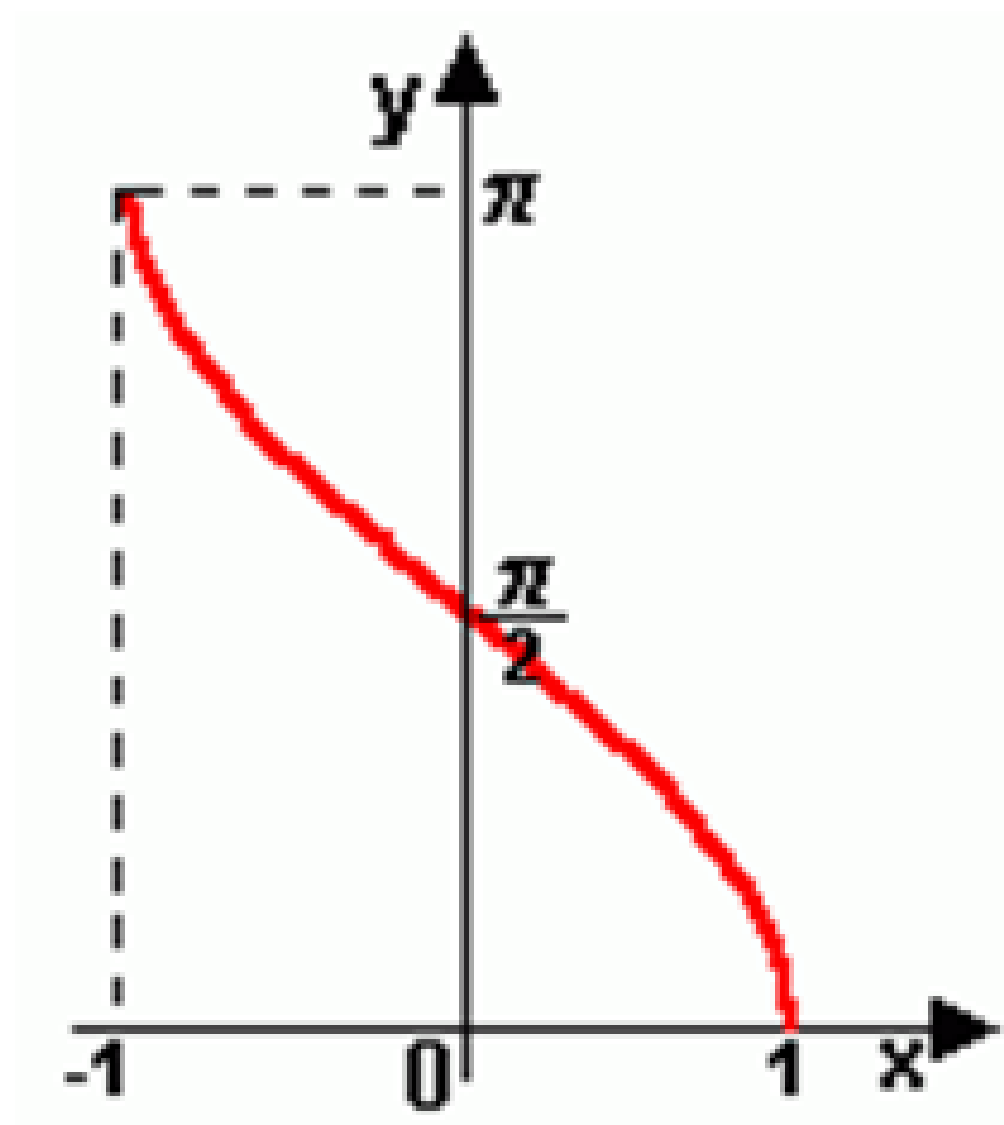
Para que uma função admita uma inversa, ela precisa ser bijetora. Ou seja, todo elemento da imagem possui um único correspondente no domínio. Isso significa que elementos diferentes no conjunto  $A$  precisam estar associados a elementos diferentes no conjunto  $B$ , ou seja, não pode haver dois ou mais elementos do conjunto  $A$  que possuem o mesmo correspondente no conjunto  $B$ .

# Funções trigonométricas inversas: arcsen(x)

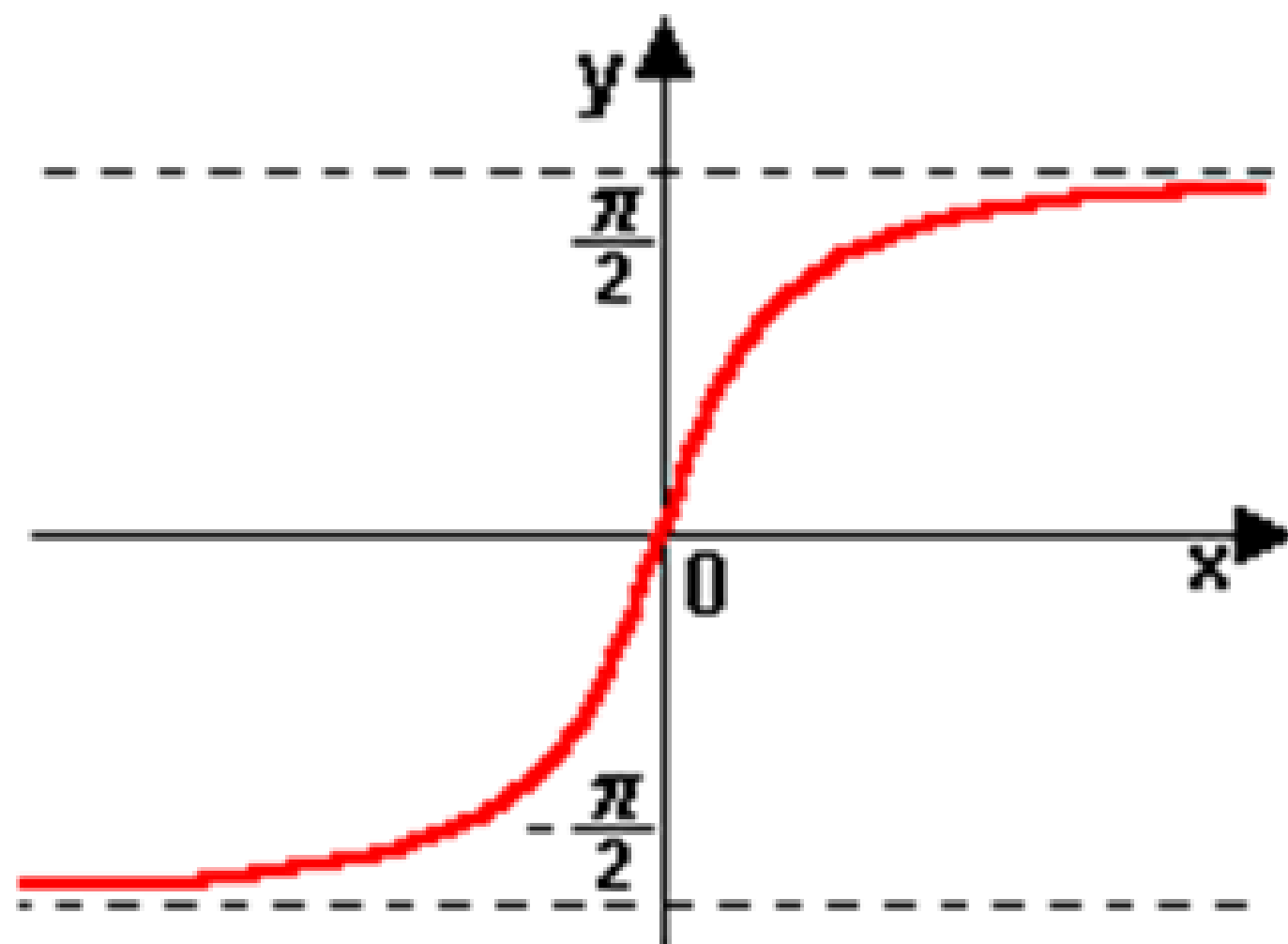




# Funções trigonométricas inversas: arccos(x)



# Funções trigonométricas inversas: arctan(x)



# DESAFIO 1



Quais soluções da equação  $\cos^2(\alpha) + \sin(\alpha) - 1 = 0$ , considerando um arco pertencente ao intervalo  $[0, 2\pi]$ ?

## DESAFIO 2



Sabendo que  $2\text{sen}(\alpha) = 3\text{tg}^2(\alpha)$ , onde  $0 < \alpha < \pi/2$ , qual o valor do  $\text{cos}(\alpha)$ ?

The background is a solid light blue color. There are four decorative triangles: one in the top-left corner (black outline, pointing down), one in the top-right corner (white fill, black outline, pointing up), one in the bottom-left corner (white fill, black outline, pointing down), and one in the bottom-right corner (black outline, pointing up).

OBRIGADA/O!